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Basic ruin models

Wojciech Bartoszek and Michał Janiak

The lecture is devoted to a simplified model of the ruin problem for a company, whose incomes are random and the costs of running are constant. We show that regardless of the distribution of the profit size from each particular contract the probability of ruin of the firm decays exponentially (or sub-exponentially in a generalized model) with the value of the initial capital. We find explicit formulae in some important cases or show how to build approximate algorithms in a general case. After a short introduction to Cramer–Lundberg risk process we proceed to the *dual risk surplus process*

$$U(t) = u + \sum_{n=1}^{N(t)} \xi_n - ct$$

which is commonly known as the dual model. Here ξ_n are iid nonnegative random variables, u denotes the initial surplus, $c > 0$ is a constant representing (unit) running costs, t stands for time and $N(t)$ is a classical Poisson process. Using functional analysis arguments we find a new short and self–contained proof for the formula of non–ruin probability i.e.

$$\psi(u) = P(\{\forall_{t \geqslant 0}\ U(t) \geqslant 0 \mid U(0) = u\}) = 1 - e^{-au} \ .$$

More realistic case (with uncertain contracts) is modeled by a generalized dual risk model with the surplus process

$$U(t) = u + \sum_{n=1}^{N(t)} \delta_n \xi_n - ct \ ,$$

where δ_n is a sequence of iid random variables with

$$P(\delta_n = -\delta) = \varepsilon \qquad \text{and} \qquad P(\delta_n = \delta) = 1 - \varepsilon$$

for some coefficients $0 < \varepsilon, \delta < 1$. Here we show (with restriction that ξ_n are exponentially distributed) that

$$1 - e^{-a_o u} \leqslant \Psi(u) \leqslant 1 - e^{-A^* u}$$

for some constants $A^* > a_o > 0$ which may be calculated efficiently from the model.

Pricing options in illiquid markets: symmetry reductions and exact solutions

Ljudmila Bordag

Empirical observations on the market in the last decade allow to draw the conclusion that the liquidity is an increasing problem for large traders. Many approaches have been developed to take into account the feedback effect of a fund hedging strategy or of the transaction costs of large traders. Most of these models are represented by nonlinear variations of the well-known Black-Scholes equation. These models contain a parameter which reflects the measure of a large traders influence on the market. Usually it is expected that the equations will reduce to the Black-Scholes one if this influence becomes small. In other words the nonlinear equations are considered as perturbations of the linear Black-Scholes equation.

From the analytical point of view these equations can be divided roughly into two classes: equations with regular and with singular perturbations. The last type equations are fully nonlinear ones and are the least of all studied by analytical methods. However, an analytical study of these models gives the possibility to find scopes of applications of different models represented by fully nonlinear partial differential equations.

We apply the Lie group analysis to these equations. The models under investigation were introduced by Bank and Baum, Frey, Frey and Streme, Sircar and Papanicolaou under different assumptions on the market. From an analytical point of view they have similar features concerning the nonlinearities. It is typical for all these equations that they possess rich Lie symmetry groups which allow an introduction of invariant variables and reduction the corresponding partial differential equation to ordinary ones. In some cases it is possible to find even exact invariant solutions to these equations.

Most of the exact solutions for a given nonlinear equation have no counterpart in the linear Black-Scholes case. They intrinsically reflect a nonlinearity of the equation. The invariant solutions can be used as benchmarks for different numerical methods or as a starting point for stability investigations. Some of these solutions approximate typical financial derivatives rather good.

Minimax asymptotic optimality of the first order for quickest detection problem of intensity change of poisson process

Evgeniy V. Burnaev

Quickest detection problem for Poisson process in minimax setting is considered. For minimax average delay time lower-bound and upper-bound estimates are obtained. It is proved that this estimates coincide asymptotically (for big values of intensity of poisson process and mean time until false alarm) if method on basis of Shiryaev-Roberts process is used.

The log processes with independent increments

Joachim Domsta

The difference between the increments of the logarithmic derivative of positive processes and the logarithm itself is discussed in details. The suitable proposals concerning the way of modelling the price of securities are discussed briefly. The main reason which makes this analysis important is that the processes for describing the logarithmic derivative cannot have negative jumps of value greater than one, unless the price process changes its sign. Therefore the problem of choosing a natural model for the process describing the interest rate becomes more complicated. The formulas given in this work can help in avoiding some serious errors. Just for simplicity, the particular formulas are related to processes with independent increments, however the main rules are applicable to a much more wide class of processes.

Quantile hedging with re-discounting on the complete finacial market

Yuliya Mishura

The problem of hedging of contingent claims is well known in the case of complete non-arbitrage models. Consider an investor, who wants to ensure that a claim H will be hedged and operates with an asset, whose price is modelled by a semimartingale. Necessary and sufficient condition for this is the availability of capital $H_0 = E_{P*}(H)$, where E_{P*} is the expectation w.r.t. the unique martingale measure $P*$, i.e. the measure w.r.t. which the price process X_t is a martingale. If investor in unwilling or unable to use whole amount H_0 and wants or is able to use certain amount $\nu < H_0$, it follows from the absence of arbitrage that he cannot supply the replication of claim H in all possible scenario, i.e. he cannot hedge the claim H with probability 1. In this case the problem of quantile hedging for investor can be reduced to maximization of success probability, i.e. the probability to hedge the claim. In paper [Follmer, Leukert] the general principles concerning such type of hedging are considered in the case, where the price process is a semimartin-gale. In paper [Krutchenko, Melnikov] the special case of jump-diffusion market is considered, the stochastic differential equation for hedging strategy is deduced, the hedging strategy and the price of European option are obtained.

In our work we move away from the semimartingale model, try to consider models with so called long-range dependence and clarify in this case the dependence of maximal possible success probability on the available initial capital $\nu < H_0$.

Risk minimizing strategies for a portfolio of interest-rate securities

Andrzej Palczewski

This talk presents an application of stochastic control methods to fixed income management. The problem is considered in the context of an incomplete market with external economic factors. The objective of the investor is the minimization of the shortfall risk. This problem is reduced to a multidimensional Bellman equation and it has been shown that for a large class of loss functions from HARA class the Hamilton-Jacobi-Bellman equation has a sufficiently smooth solution. This solution guaranties the existence of well defined investment strategy. A special example of bond portfolio with interest rate governed by the

Gaussian HJM model has been explicitly solved. Numerical simulations for the CIR model will be also presented.

Haar interpolation of financial markets

Igor I. Pavlov

On finite security markets we consider two properties of martingale measures: Haar uniqueness property (HUP) and universal Haar uniqueness property (UHUP). Results on the existence of such measures and on the coinciding of the totality of all such measures with the set of all martingale measures will be presented. With the help of martingale measures satisfying HUP or UHUP we can extend initial arbitrage-free (B,S)-market to arbitrage-free and complete ones. That will be made by introducing of additional time moments and corresponding Haar filtrations. The theory developed in this way will be applied to several special financial markets so as to construct perfect hedges for arbitrary contingent claims.

Recurrence formulas for price bounds of contingent claims in finite discrete time

Dmitry Rokhlin

We present recurrence formulas for price bounds of contingent claims in a general foreign exchange market model with transaction costs. As examples, we consider the frictionless market model and the model with non-zero bid-ask spreads under various assumptions: finite probability space, bounded and unbounded returns. The main result is a non-trivial consequence of the martingale selection theorem.

Properties of optimal exercise domain for futures exchange contract

Georgiy Shevczenko

Let $S1(t)$, $S2(t)$ be correlated geometric Brownian motions. We consider the maximization problem $\max_\tau E[S1(\tau) - S2(\tau)]$, where τ is a stopping time from $[0, T]$. This problem arises naturally from the problem of optimal exercise of a futures contract to exchange two correlated assets in the Black-Scholes market model. To solve this problem is crucial for purposes of strategic investment, which usually uses offsetting positions in correlated assets to hedge its losses.

A similar problem, but on infinite time interval, was studied in [2], and in [1], where a multi-asset generalization is also considered. For a finite time horizon, the problem gets considerably more complicated and cannot be solved explicitly. In this paper we study generic properties of the optimal stopping set and its boundary curve, and derive an integral equation for the latter.

References

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Toward to the optimality of the rule "Buy-and-Hold" in the Financial Industry

Albert N. Shiryaev

We consider (B, S)-market (Black–Scholes):

$$dB_t = rB_t\,dt, \qquad B_0 = 1,$$
$$dS_t = S_t(a\,dt + \sigma\,dW_t), \qquad S_0 = 1,$$

where $W = (W_t)_{t\geqslant 0}$ is a Wiener process. Let $T < \infty$ and

$$P_t = \frac{S_t}{B_t}, \qquad M_T = \max_{0\leqslant t\leqslant T} P_t$$

and let $U = U(x)$ be a utility function.

In our talk we are interested in finding an optimal stopping time τ^* (i.e., time of selling stock) such that

$$\sup_{0\leqslant\tau\leqslant T} \mathsf{E}\,U\left(\frac{P_\tau}{M_T}\right) = \mathsf{E}\,U\left(\frac{P_{\tau^*}}{M_T}\right).$$

Growth and risk sensitive optimal portfolios under proportional transaction costs with obligatory diversification

Łukasz Stettner

We consider portfolio selection problems with growth and risk sensitive optimal criterion over infinite time horizon with proportional transaction costs, consisting of a part proportional to the volume of transaction and fixed proportional, considered as management costs. This kind of problems can be reduced to an impulsive control with average cost per unit time criterion or impulsive control with multiplicative cost functional of the portions of capital invested in assets. Since such cost functional may lead to trivial strategies - location of the whole capital in one asset, we impose an obligatory diversification pressing us to change portfolio, whenever the portion of capital invested in any asset is too large or too small. As a result the process of portions of capital invested in assets has a nice ergodic properties and we are able to solve suitable Bellman equations corresponding to control problems.