G E N E R A L   P H Y S I C S

Bohm's Gedankenexperiment- the relativistic version.

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ABSTRACT
The singlet state of a pair of Dirac electrons is constructed. The Bell Theorem cannot be proved without the subsidiary postulate of correspondence.

1. INTRODUCTION

The Gedankenexperiment proposed by Bohm [1] as the new version of famous EPR paradox became the starting point for the origınal proof of the Bell theorem [2]. Although the Bell ´s theorem was formulated later in the relativistic version for the case of correlated photons, the question of validity of Bohm´s considerations within the framework of relativistic quantum theory is still interesting. The suspition about a conceptual nontriviality of the problem has been suggested by the fact that spin is not an observable in the Poincaré-invariant theories [3]. An attempt of getting clear of that trouble belongs to the same class of problems as the question of Dirac electron's velocity. As will be shown, the solution proposed in some standard text-books [4] /a restriction of observables to even operators/ brings new questions. The motivation for writing the text was the author´s will of convincing himself about the value of EPR considerations for electrons.

2. The system under the consideration consists of a pair of space-like-separated electrons. Let $\vec{a}$ and $\vec{b}$ be arbitrary unit vectors and $\vec{t}$, $\vec{n}$, $\vec{k}$ span orthonormal bases for the reference frames of both particles. By definition $\vec{t} = \vec{p}/|\vec{p}|$, $\vec{n}\cdot\vec{t} = 0$ and $\vec{k}=\vec{t}\times\vec{n}$ / $\vec{p}$ is a momentum of one of those particles/. Let us also introduce a spinor $\omega_\pm$ satysfying relations

$$\vec{t}\vec{\sigma}\,\omega_\pm = \pm\,\omega_\pm\,,\quad \vec{n}\vec{\sigma}\,\omega_\pm = \omega_\mp\,,\quad \vec{k}\vec{\sigma}\,\omega_\pm = \pm i\,\omega_\mp$$

The spin projection operator is

$$S_3^{ab} = \vec{a}\cdot\vec{S}\otimes 1\!\!1 + 1\!\!1\otimes\vec{b}\cdot\vec{S}$$

and the square of total spin is $(\vec{S}^{ab})^2 = \vec{S}^2\otimes 1\!\!1 + 1\!\!1\otimes\vec{S}^2 +$

$$+\, 2\,\vec{t}\vec{S}\otimes\vec{t}\vec{S} + 2\,\vec{n}\vec{S}\otimes\vec{n}\vec{S} + 2\,\vec{k}\vec{S}\otimes\vec{k}\vec{S}$$

At the nonrelativistic case $(\vec{S}^{ab})^2$ written in a basis of S_3^{ab} is a matrix

$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

The middle box has eigenvalues 2 and 0,with corresponding eigen-vectors

$$\Psi_{10} = \frac{1}{\sqrt{2}} \left(\Psi_+ \otimes \Psi_- + \Psi_- \otimes \Psi_+ \right)$$

and

$$\Psi_{00} = \frac{1}{\sqrt{2}} \left(\Psi_+ \otimes \Psi_- - \Psi_- \otimes \Psi_+ \right)$$

The average value of an observable $2S_3^a \cdot 2S_3^b$ in a singlet state is

$$P(\vec{a},\vec{b}) = \langle \Psi_{00} | \vec{a}\vec{\sigma} \otimes \vec{b}\vec{\sigma} | \Psi_{00} \rangle = -\vec{a}\cdot\vec{b}$$

Since for any measurement that observable gives ± 1, $P(\vec{a},\vec{b})$ should satysfy the Bell inequality.The violating of the inequality by $P(\vec{a},\vec{b})$ leads to the known theorem of nonrelativistic QM.

3.THE RELATIVISTIC CASE

By a strict analogy to Sec.2 we choose the helicity states to be the common basis for S_3^{ab} and $(\vec{S}^{ab})^2$ .In the standard representation

$$\vec{S} = \frac{1}{2}\begin{pmatrix} \vec{\sigma}, & 0 \\ 0, & \vec{\sigma} \end{pmatrix}$$

and

$$\Psi_\pm = \frac{1}{\sqrt{2\varepsilon}}\begin{pmatrix} \sqrt{\varepsilon+m} & \omega_\pm \\ \pm\sqrt{\varepsilon-m} & \omega_\pm \end{pmatrix}$$

where $\qquad \varepsilon = \sqrt{\vec{p}^{\,2} + m^2}$

$\Psi_\pm$ satysfy orthogonality conditions

$$\Psi_i^+ \Psi_j = \delta_{ij}, \quad \bar{\Psi}_i \Psi_j = \frac{m}{\varepsilon}\delta_{ij}$$

with $\bar{\Psi}_i = \Psi_i^+ \gamma^0$ and $i,j = +,-$ .The (8x8) matrix $(\vec{S}^{ab})^2$ between the states $\Psi_i \otimes \Psi_j$ and $\bar{\Psi}_m \otimes \bar{\Psi}_n$ is

$$\begin{pmatrix} 2\frac{m^2}{\varepsilon^2}\mathbb{1} & 0 & 0 & 0 \\ 0 & \frac{m^2}{\varepsilon^2}\mathbb{1} & \mathbb{1} & 0 \\ 0 & \mathbb{1} & \frac{m^2}{\varepsilon^2}\mathbb{1} & 0 \\ 0 & 0 & 0 & 2\frac{m^2}{\varepsilon^2}\mathbb{1} \end{pmatrix}$$

Its eigenvalues are

$$\lambda_{12} = 2\,\frac{m^2}{\varepsilon^2}\,, \quad \lambda_3 = \frac{m^2}{\varepsilon^2} + 1\,, \quad \lambda_4 = \frac{m^2}{\varepsilon^2} - 1$$

$\lambda_4 \leqslant 0$ corresponds to the nonrelativistic singlet state and

$$\Psi_4 = \frac{1}{\sqrt{2}}\left(\Psi_+ \otimes \Psi_- - \Psi_- \otimes \Psi_+\right)$$

The analogous average is

$$\langle \Psi_4 | 2\,S_3^a \cdot 2\,S_3^b | \Psi_4 \rangle = -\left(\vec{a}_\parallel \cdot \vec{b}_\parallel + \frac{m^2}{\varepsilon^2}\,\vec{a}_\perp \cdot \vec{b}_\perp\right)$$

Despite appearances this function cannot be used for the construction of Bell theorem. It follows from the fact that $\vec{a}\,\vec{S} \otimes \vec{b}\,\vec{S}$

is not an observable i.e. does not commute with Hamiltonian because of Spinbewegung oscillations [3]. The Spinbewegung vanishes if we restrict ourselves to those parts of operators that have no matrix elements between states with different signs of the energy.

One can try to motivate this operation "physically" by requiring that a particle cannot transform to an antiparticle and vice versa. On the other hand one should be aware of the fact, that the requirement does not result from the theory but is added from outside to keep common sense character of predictions. In the former paper [5] the author has shown that the time dependance of spin operator possesses a fundamental importance for the structure of fields equations. And all the time dependance of spin is contained in a Spinbewegung. So at the ground of Dirac theory the most ~~consequent~~ consistent would be the statement that Bohm's Gedankenexperiment does not have a relativistic equivalent.
For the sake of completness let us investigate the alternative case for which the fundamental is the following

Postulate of correspondence

To measurable physical quantities correspond Hermitian even operators.

Let us now introduce the projectors $\Gamma_{\pm} = \frac{1}{2}(\mathbb{1} \pm \Lambda)$,

$$\Lambda = H / \sqrt{H^2}$$

Any operator A can be split to $A = A_{even} + A_{odd}$

$$A_{even} = \Gamma_+ A \Gamma_+ + \Gamma_- A \Gamma_-$$

$$A_{odd} = \Gamma_+ A \Gamma_- + \Gamma_- A \Gamma_+$$

As one can see A_{even} vanishes between the states of opposite energy. For Dirac spin

$$\vec{S}_{ev} = (\mathbb{1} - p^2/\varepsilon^2)\vec{S} + \frac{p^2}{\varepsilon^2}(\vec{t}\cdot\vec{S})\vec{t} + \frac{imp}{2\varepsilon^2}\vec{t}\times\vec{\gamma}$$

then

$$\vec{t}\,\vec{S}_{ev} = \vec{t}\,\vec{S}$$

$$\vec{n}\,\vec{S}_{ev} = \frac{m^2}{\varepsilon^2}\vec{n}\cdot\vec{S} - \frac{imp}{2\varepsilon^2}\vec{k}\cdot\vec{\gamma}$$

$$\vec{k}\,\vec{S}_{ev} = \frac{m^2}{\varepsilon^2}\vec{k}\,\vec{S} + \frac{imp}{2\varepsilon^2}\vec{n}\cdot\vec{\gamma}$$

For any $\vec{a}$ $\vec{a}\vec{S}$ commutes with H .Its eigenfunctions are

$$\psi_{\pm}^{\vec{a}} = N\begin{pmatrix} \sqrt{\varepsilon+m}\,\left[\left(|\lambda_{\vec{a}}| + \frac{\vec{a}\cdot\vec{t}}{2}\right)\omega_{\pm} \pm \frac{m\vec{a}\cdot\vec{n}}{2\varepsilon}\omega_{\mp}\right] \\ \sqrt{\varepsilon-m}\,\left[\pm\left(|\lambda_{\vec{a}}| + \frac{\vec{a}\cdot\vec{t}}{2}\right)\omega_{\pm} - \frac{m\vec{a}\vec{n}}{2\varepsilon}\omega_{\mp}\right] \end{pmatrix}$$

and the eigenvalues are

$$\lambda_{\vec{a}} = \pm\frac{1}{2}\,\frac{\sqrt{(\vec{p}\cdot\vec{a})^2 + m^2}}{\varepsilon}$$

Our task reduces now to finding the "singlet" state and counting the average value of the operator

$$\vec{a}\cdot\vec{S}_{ev}/|\lambda_{\vec{a}}| \otimes \vec{b}\cdot\vec{S}_{ev}/|\lambda_{\vec{b}}|$$

The "single particle spin " satysfies the following commutation algebra

$$[\vec{n}\vec{S}_{ev}, \vec{k}\vec{S}_{ev}] = i\,\frac{m^2}{\varepsilon^2}\,\vec{t}\cdot\vec{S}$$

$$[\vec{t}\vec{S}, \vec{n}\vec{S}_{ev}] = i\,\vec{k}\vec{S}_{ev}$$

$$[\vec{k}\vec{S}_{ev}, \vec{t}\vec{S}] = i\,\vec{n}\vec{S}_{ev}$$

Moreover

$$\vec{t}\vec{S}\,\psi_{\pm} = \pm\frac{1}{2}\,\psi_{\pm}$$

$$\vec{n}\vec{S}_{ev}\,\psi_{\pm} = \frac{m}{2\varepsilon}\,\psi_{\mp}$$

$$\vec{k}\vec{S}_{ev}\,\psi_{\pm} = \pm i\,\frac{m}{2\varepsilon}\,\psi_{\mp}$$

$$\psi_{\pm} \equiv \psi_{\pm}^{\vec{t}}$$

A problem appears when one tries to define the square of spin.

It comes from the evenness of $\vec{S}^2$ .On the other hand the ~~consequent~~ consistent description requires a choice of $(\vec{S}_{ev})^2$ .But

$$(\vec{S}_{ev})^2 = \vec{S}^2 - \frac{p^2}{2\varepsilon^2}\,\mathbb{1}$$

It is the hardest shortcoming of an "even formalism".Luckily in the two particle case $(\vec{S}\otimes \mathbb{1} + \mathbb{1}\otimes\vec{S})^2$ is not even.

So it is necessary to choose

$$(\vec{S}_{ev}\otimes \mathbb{1} + \mathbb{1}\otimes\vec{S}_{ev})^2 .$$

In the helicity basis it is ~~necessary~~ the 16x16 matrix

$$\begin{pmatrix} (2-\frac{p^2}{\varepsilon^2})\mathbb{1} & 0 & 0 & 0 \\ 0 & (1-\frac{p^2}{\varepsilon^2})\mathbb{1}, & \frac{m^2}{\varepsilon^2}\mathbb{1} & 0 \\ 0 & \frac{m^2}{\varepsilon^2}\mathbb{1} & ,\ (1-\frac{p^2}{\varepsilon^2})\mathbb{1} & 0 \\ 0 & 0 & 0 & (2-\frac{p^2}{\varepsilon^2})\mathbb{1} \end{pmatrix}$$

Its eigenvalues : $\lambda_{12} = 2 - \frac{p^2}{\varepsilon^2}$, $\lambda_3 = 1 - \frac{p^2 - m^2}{\varepsilon^2}$

$$\lambda_4 = 0 .$$

Singlet : $\Psi_4 = \frac{1}{\sqrt{2}}\left(\Psi_+ \otimes \Psi_- - \Psi_- \otimes \Psi_+\right)$.

Finally

$$p(\vec{a},\vec{b},\vec{p}) = \langle \Psi_4 |\, \vec{a}\,\vec{S}_{ev}/|\lambda_{\vec{a}}| \otimes \vec{b}\,\vec{S}_{ev}/|\lambda_{\vec{b}}|\, |\Psi_4\rangle =$$

$$= -\frac{1}{4|\lambda_{\vec{a}}\lambda_{\vec{b}}|}\left(\vec{a}_{\parallel}\cdot\vec{b}_{\parallel} + \frac{m^2}{\varepsilon^2}\,\vec{a}_{\perp}\cdot\vec{b}_{\perp}\right)$$

So the relativistic electrons obeying the Postulate remain a non-local system. It becomes especially visible when one choses $\vec{a}$ and $\vec{b}$ perpendicular to $\vec{p}$. Then

$$P(\vec{a}, \vec{b}, \vec{p}) = - \vec{a} \cdot \vec{b}$$

and that obviously violates Bell´s inequality.

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